INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, Second Semester, 2014-15
Statistics - II, Midterm Examination, March 5, 2015
Answer all questions

1. Let X_1 and X_2 be two independent observations from the $\mathrm{Exp}(\lambda)$ distribution with $\lambda > 0$.
(a) Show that $\frac{X_1}{X_1+X_2}$ is an ancillary statistic.
(a) Show using Basu's theorem that $\frac{X_1}{X_1+X_2}$ and $X_1 + X_2$ are independently distributed. [10]

2. Suppose $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ are independent random samples, respectively, from $N(2\mu, 10^2)$ and $N(\mu, 5^2)$, where $-\infty < \mu < \infty$ is the unknown parameter of interest.
(a) What is the probability distribution of $T = (\bar{X} + 2\bar{Y})/4$?
(b) Is T sufficient for μ?
(c) Find minimal sufficient statistic for μ. Is it complete?
(d) Find the MLE and UMVUE of μ. [20]

3. Let $X_1, \ldots, X_n$ be a random sample from $\mathrm{Poisson}(\lambda)$, $\lambda > 0$.
(a) Find the Fisher information of λ contained in the random sample.
(b) Find the Cramer-Rao lower bound on the variance of an unbiased estimator of $q(\lambda) = \exp(-\lambda)$.
(c) Find the UMVUE of $q(\lambda)$. Does it attain the lower bound given in (b) above? [15]

4. Consider a random sample $X_1, X_2, \ldots, X_n$ from $U(\theta, \theta + 1)$ where $-\infty < \theta < \infty$. Construct a 95% confidence interval for θ which has the form:
$[X_{(n)} - (1 - c_2), X_{(1)} - c_1]$ for some constants c_1 and c_2. [5]